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# DISCRETE MATHEMATICS CURRICULUM

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ACKNOWLEDGEMENTS

The Discrete Mathematics Curriculum guide was developed for Neptune High School through the efforts of Nora Healey, Michael Matson, and Rodney Taylor, Neptune High School mathematics teachers, in cooperation with Nancy Moore-Fuss, supervisor, and under the guidance of Edward Dever, Director of Curriculum and Instruction.

The teachers are to be commended for their dedication in formatting this curriculum into UbD and their expertise in the area of Discrete Mathematics. This curriculum guide goes beyond basic skill instruction, and includes a variety of mathematical models which devote a greater percentage of instructional time to problem-solving and active learning. It is our hope that this guide will serve as a valuable resource for the staff members who teach this course and that they will feel free to make recommendations for its continued improvement.

The Discrete Mathematics Curriculum guide was written with related pacing guide and assessments in alignment to the 2009 New Jersey Core Curriculum Content Standards for Mathematics, Language Arts Literacy, 21st Century Life and Careers, and Technology.
NEPTUNE TOWNSHIP SCHOOL DISTRICT

DISTRICT
MISSION STATEMENT

The primary mission of the Neptune Township School System is to prepare all students for life in the 21st Century by encouraging them to recognize that learning is a continuing process. Thus, it is with high expectations that our schools foster:

- A strong foundation in academic areas, modern technologies, life skills and the arts.
- A positive and varied approach to teaching and learning.
- An emphasis on critical thinking skills and problem-solving techniques.
- A respect for an appreciation of our world, its resources and its peoples.
- A sense of responsibility, good citizenship and accountability.
- An involvement by the parents and the community in the learning process.
Neptune Township School District
Educational Outcome Goals

The students in the Neptune Township schools will become life-long learners and will:

- Become fluent readers, writers, speakers, listeners, and viewers with comprehension and critical thinking skills.
- Acquire the mathematical skills, understandings, and attitudes that are needed to be successful in their careers and everyday life.
- Understand fundamental scientific principles, develop critical thinking skills, and demonstrate safe practices, skepticism, and open-mindedness when collecting, analyzing, and interpreting information.
- Become technologically literate.
- Demonstrate proficiency in all New Jersey Core Curriculum Content Standards (NJCCCS).
- Develop the ability to understand their world and to have an appreciation for the heritage of America with a high degree of literacy in civics, history, economics and geography.
- Develop a respect for different cultures and demonstrate trustworthiness, responsibility, fairness, caring, and citizenship.
- Become culturally literate by being aware of the historical, societal, and multicultural aspects and implications of the arts.
- Demonstrate skills in decision-making, goal setting, and effective communication, with a focus on character development.
- Understand and practice the skills of family living, health, wellness and safety for their physical, mental, emotional, and social development.
- Develop consumer, family, and life skills necessary to be a functioning member of society.
- Develop the ability to be creative, inventive decision-makers with skills in communicating ideas, thoughts and feelings.
- Develop career awareness and essential technical and workplace readiness skills, which are significant to many aspects of life and work.
MATHEMATICS DEPARTMENT
PROGRAM PHILOSOPHY

The primary goal of the Mathematics Department is to enable ALL of our students to acquire the mathematical skills, understandings, and attitudes that they will need to be successful in their careers and daily lives. To compete in today’s global, information-based economy, students must be able to solve real problems, reason effectively, and make logical connections. Jobs increasingly require mathematical knowledge and skills in areas such as data analysis, problem-solving, pattern recognition, statistics and probability.

To reach our goal, mathematics educators in Neptune Township Schools will provide our students with a learning environment which is characterized by the following:

- Students excited by and interested in their activities.
- Students learning important mathematical concepts rather than simply memorizing and practicing procedures.
- Students posing and solving meaningful problems.
- Students working together to learn mathematics.
- Students writing and talking about math topics every day.
- Students using calculators and computers as important tools of learning.
- Students whose teachers who have high expectations for ALL of their students.
- Students being assessed by a variety of assessment strategies, not just traditional short-answer tests.

By providing our students with experiences that will enable them to be successful learners of mathematics, we will also provide our graduates with the skills and abilities they will need to compete in the global marketplace of the 21st Century.
DISCRETE MATHEMATICS

COURSE DESCRIPTION

The world has become a more complex place. While economically competitive nations have long recognized the value of mathematics learning and have relied on a citizenry who is mathematically literate, the context of life and work in a knowledge economy, heavily dependent on rapidly emerging technologies, demands the kinds of logical reasoning, problem solving, analytical and abstract thinking skills uniquely acquired through the study of mathematics. The development of such skills by all students is possible when grounded in the philosophy that success in mathematics is not solely limited to students that possess ‘math talent’, but rather on the belief, that with effort, all students can learn and effectively use mathematics (The National Mathematics Advisory Panel Report, 2008).

The Discrete Mathematics course attempts to bring the excitement of contemporary mathematical thinking to the nonspecialist. In science and industry, mathematical models are the main tools for analyzing and solving problems that arise. Our aim is to convey to the student the power of mathematics by showing the great variety of problems that can be modeled and solved by quantitative means. In traditional mathematics courses, students spend a lot of time learning the tools of mathematics—how to manipulate symbols, how to solve equations. In this course, students will spend time learning the uses of mathematics and the power of mathematics to help us to understand so many different parts of our everyday lives and the world itself.

Prerequisite: completion of Geometry and Algebra II
DISCRETE MATHEMATICS
CURRICULUM

INSTRUCTIONAL RESOURCES

TITLE: For All Practical Purposes, 8th ed.
PUBLISHER: W. H. Freeman and Company
DATE: 2009
AUTHOR(S): S. Garfunkel, Project Director

ADDITIONAL MATERIALS:

Excursions in Modern Mathematics, 6th ed.
Pearson Prentice Hall
2007
P. Tannenbaum

W. H. Freeman and Company
2006
N. Crisler and G. Froelich

COMAP Modules
The Mathematics of Conflicts
Problem Solving Using Graphs
Codes Galore
The Mathematician’s Coloring Book

Leadership Program in Discrete Mathematics
Rutgers University

TECHNOLOGY: Donald in MatheMagicLand—video

Websites
Discrete Mathematics Project
http://www.colorado.edu/education/DMP

http://math.rice.edu/

Numb3rs Activities Seasons 1-3
http://www.ti.com

Appelet exercises from For All Practical Purposes (FAPP)
http://whfreeman.com/fapp8e
Established Goals:
NJCCCS 4.4—All students will be able to collect, organize, and display relevant data to answer questions that can be addressed with data; use appropriate statistical methods to develop and evaluate inferences and predictions that are based on data; and apply basic concepts of probability in order to make informed choices and reasonable decisions about information presented.
NJCCCS 4.5—All students will use the mathematical processes of problem solving, communication, connections, reasoning, representations and technology to gain deeper understanding of mathematical concepts, problem solving and how to communicate mathematical ideas.
NJCCCS 3.2—All students will write in clear, concise, organized language that varies in content and form for different audiences and purposes.
NJCCCS 8.1—All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaboratively to create and communicate knowledge.

Understandings:
Students will understand that...
- Probability quantifies the likelihood that something will happen and enables us to make predictions and informed decisions.
- Experimental results tend to approach theoretical probabilities after a large number of trials.
- Grouping by attributes (classification) can be used to answer mathematical questions.
- Algorithms can effectively and efficiently be used to quantify and interpret discrete information.

Essential Questions:
- How can experimental and theoretical probabilities be used to make predictions or draw conclusions?
- How can attributes be used to classify data/objects?
- What counting strategy works best in a given situation?

Students will know…
- the fundamental counting principle.
- order is important in permutations
- order is not important in combinations.
• the “law of large numbers” as it pertains to experimental probability.
• the difference between dependent and independent events.
• which events are considered conditional and which are complementary.
• methods for finding the theoretical probability of an event.
• concepts and formulas of area.

**Students will be able to…**

• distinguish between independent and dependent events and select the appropriate method to compute the probabilities in each case.
• show the sample space (the set of all possible outcomes) for compound events in an organized way (tree diagrams and organized lists) and express the theoretical probability of each outcome.
• model event simulations (spinners, dice, coins) using random number calculators.
• determine probabilities of conditional and compound events.
• determine probabilities for dependent and independent events for small sample spaces.
• analyze probability-based games with regard to expected value and fairness.
• use combinatorics to compute probabilities of compound events.
• use conditional probability to solve problems.
• use concepts and formulas of area to calculate geometric probabilities.
• estimate probabilities and make predictions based on experimental and theoretical probabilities.
STAGE 2 – Assessment Evidence

Performance Tasks:
- “A Guaranteed Millionaire” activity from resource binder.
- “Dart Board”, p. 738, Algebra II
- “LOTTO Winners” activity from the Discrete Mathematics Project at www.colorado.edu/education/DMP

Other Evidence:
- Applet exercises from FAPP, Ch. 8 found at www.whfreeman.com/fapp8e
- Writing projects from FAPP, Ch. 8, p.281.
- Discrete Probability Activities from the Discrete Mathematics Project at www.colorado.edu/education/DMP
- Counting Techniques Activities from Discrete Mathematics Project at www.colorado.edu/education/DMP
- Class work, homework, quizzes, tests.

STAGE 3 – Learning Plan

Learning Activities:
- games and probability exercises from the resource binder (section 14)
- “Fascinating But Funky Functions” from the resource binder.
- rolling dice, shooting baskets or spinning spinners to compare theoretical and experimental probabilities.
- Use a standard deck of playing cards to find how many different 5-card hands are possible and the probability of having all-five cards of the same suit.
- Sample packages of M & Ms to predict the probability of choosing a particular color. Write a letter to Hershey’s to persuade them to add a new color for M & Ms.
- practice exercises from Algebra II, Ch. 10.
- practice exercises from FAPP, Ch. 8
- “What is the Probability Cards” from resource binder.
- NUMB3RS Activities found at www.ti.com
Established Goals:
NJCCCS 4.2—All students will develop spatial sense and the ability to use geometric properties, relationships, and measurement to model, describe, and analyze phenomena.
NJCCCS 4.4—All students will be able to collect, organize, and display relevant data to answer questions that can be addressed with data; use appropriate statistical methods to develop and evaluate inferences and predictions that are based on data; and apply basic concepts of probability in order to make informed choices and reasonable decisions about information presented.
NJCCCS 4.5—All students will use the mathematical processes of problem solving, communication, connections, reasoning, representations and technology to gain deeper understanding of mathematical concepts, problem solving and how to communicate mathematical ideas.
NJCCCS 3.2—All students will write in clear, concise, organized language that varies in content and form for different audiences and purposes.
NJCCCS 8.1—All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaboratively to create and communicate knowledge.

Understandings:
Students will understand that...
- Discrete mathematics consists of tools and strategies for representing, organizing, and interpreting non-continuous data.
- Optimization is finding the best solution within given constraints.
- Grouping by attributes (classification) can be used to answer mathematical questions.
- Algorithms can effectively and efficiently be used to quantify and interpret discrete information.

Essential Questions:
- How can visual tools such as networks be used to answer real-life problems?
- How can you determine if you have found the best solution in a particular situation?
- How can efficiency be improved in the delivery of goods?

Students will know…
- the characteristics of a variety of routing problems.
- vertex-edge graphs can be used to model real-life situations.
- Euler’s Theorem.
- relevant vocabulary: path, circuit, degree(valance), vertex-edge graph

<table>
<thead>
<tr>
<th>Students will be able to…</th>
</tr>
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<tbody>
<tr>
<td>* describe the elements of a vertex-edge graph using terms such as, edges, vertices, nodes, neighbors and count the possible paths between any two nodes of a simple vertex-edged graph.</td>
</tr>
<tr>
<td>* describe vertex-edge graphs with regard to “degree of a vertex” and “connectedness”.</td>
</tr>
<tr>
<td>* develop an algorithm using a vertex-edge graph to find a circuit that includes every edge in the graph just once (e.g., snow plow or delivery routes).</td>
</tr>
<tr>
<td>* model a real-life situation with a vertex-edge graph.</td>
</tr>
<tr>
<td>* identify paths and circuits within a graph.</td>
</tr>
<tr>
<td>* use Euler’s Theorem to efficiently route delivery services through a given area in the most efficient manner.</td>
</tr>
</tbody>
</table>
### STAGE 2 – Assessment Evidence

**Performance Tasks:**
- *Domino Project*—Create a set of dominoes that meets the following criteria: 1) has no circuit and no path; 2) has a path but no circuit; 3) has a circuit.

**Other Evidence:**
- Applet exercises from FAPP, Ch. 1 found at www.whfreeman.com/fapp8e
- Writing projects from FAPP, Ch. 1, p. 29.
- Graph Theory Activities from the Discrete Mathematics Project at www.colorado.edu/education/DMP
  - Class work, homework, quizzes, tests.

### Stage 3 – Learning Plan

**Learning Activities:**
- Resource Book (Section 2) from resource binder.
- "New Math Vocabulary" activity from resource binder.
- "Muddy City Problem #1" from resource binder.
- "Office Complex Floor Plan" activity from resource binder.
- Koenigsberg Bridge problem from resource binder.
- New York City bridge problem from resource binder.
- Paris bridge problem from resource binder.
- practice exercises, pp. 188-193 from resource binder.
- practice exercises from FAPP, Ch. 1.
- NUMB3RS Activities found at www.ti.com
Established Goals:
NJCCCS 4.2—All students will develop spatial sense and the ability to use geometric properties, relationships, and measurement to model, describe, and analyze phenomena.
NJCCCS 4.4—All students will be able to collect, organize, and display relevant data to answer questions that can be addressed with data; use appropriate statistical methods to develop and evaluate inferences and predictions that are based on data; and apply basic concepts of probability in order to make informed choices and reasonable decisions about information presented.
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Understandings:
Students will understand that...
• Discrete mathematics consists of tools and strategies for representing, organizing, and interpreting non-continuous data.
  • Optimization is finding the best solution within given constraints.
  • Grouping by attributes (classification) can be used to answer mathematical questions.
  • Algorithms can effectively and efficiently be used to quantify and interpret discrete information.

Essential Questions:
• How can visual tools such as networks be used to answer real-life problems?
• How can you determine if you have found the best solution in a particular situation?
• How can efficiency be improved in the delivery of goods?

Students will know…
• how to identify Hamiltonian circuit problems and strategies for solving them.
• the Traveling Salesman Problem and strategies for solving it.
• the Nearest Neighbor Algorithm.
• the Brute Force Algorithm.
• relevant vocabulary: Traveling Salesman Problem (TSP), cheapest link, complete graph, Hamilton circuit, Nearest Neighbor, optimal route, weighted graph

**Students will be able to...**

• use vertex-edge graphs or maps and develop an algorithm to answer practical problems, such as, finding the shortest delivery route that stops at specified places, or the shortest travel route from one site on a map to another.
• develop an algorithm using a vertex-edge graph to find the shortest circuit that includes every vertex of the graph (e.g., TSP).
• model a real-world map with a vertex-edge graph.
• identify a Hamilton circuits within a graph.
• use a variety of methods to find an optimal solution to TSP.
STAGE 2 – Assessment Evidence

**Performance Tasks:**
- “The Great Kaliningrade Circus” activity from resource binder, p. 230 A.
- “Tendency To Be Tardy” activity from resource binder (Discrete Mathematics Project)

**Other Evidence:**
- Applet exercises from FAPP, Ch. 2 found at [www.whfreeman.com/fapp8e](http://www.whfreeman.com/fapp8e)
- Writing projects from FAPP, Ch. 2, p. 65.
- Graph Theory Activities from the Discrete Mathematics Project at [www.colorado.edu/education/DMP](http://www.colorado.edu/education/DMP)
- Class work, homework, quizzes, tests.

Stage 3 – Learning Plan

**Learning Activities:**
- “What’s the Shortest Route” activity from resource binder.
- “Jack and Jill’s Lanterns” activity from resource binder.
- “Shortest Route from Mt. Vernon to Aurora” from resource binder.
- “Family Vacation” activity from resource binder.
- “The Traveling Sales Representative” activity from resource binder.
- “Fire Trucks” activity from resource binder.
- Resource Book (section 3) on Hamilton Circuits from resource binder.
- practice exercises, pp. 221-230 from resource binder.
- practice exercises from FAPP, Ch. 2.
- NUMB3RS Activities found at [www.ti.com](http://www.ti.com)
Established Goals:
NJCCCS 4.2—All students will develop spatial sense and the ability to use geometric properties, relationships, and measurement to model, describe, and analyze phenomena.
NJCCCS 4.4—All students will be able to collect, organize, and display relevant data to answer questions that can be addressed with data; use appropriate statistical methods to develop and evaluate inferences and predictions that are based on data; and apply basic concepts of probability in order to make informed choices and reasonable decisions about information presented.
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NJCCCS 8.1—All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaboratively to create and communicate knowledge.

Understandings:
Students will understand that...
• Discrete mathematics consists of tools and strategies for representing, organizing, and interpreting non-continuous data.
  • Optimization is finding the best solution within given constraints.
  • Grouping by attributes (classification) can be used to answer mathematical questions.
  • Algorithms can effectively and efficiently be used to quantify and interpret discrete information.

Essential Questions:
• How can visual tools such as networks be used to answer real-life problems?
• How can you determine if you have found the best solution in a particular situation?

Students will know…
• trees are specialized vertex-edge graphs.
• minimum spanning trees can be used to solve minimum network problems.
• strategies to find the shortest network.
  • relevant vocabulary: junction point, spanning tree, minimum spanning tree, shortest network, tree

Students will be able to…
  • differentiate graphs that are trees from those that are not trees.
  • find a minimum cost spanning tree in a graph.
  • find the shortest connection between cities.
  • find junction points within graphs.
### STAGE 2 – Assessment Evidence

**Performance Tasks:**
- “Happyville Rail System” activity from resource binder, p. 265, #60.
- “Picking Up the Pieces” activity from resource binder (Discrete Mathematics Project)

**Other Evidence:**
- Applet exercises from FAPP, Ch. 2 found at [www.whfreeman.com/fapp8e](http://www.whfreeman.com/fapp8e)
- Writing projects from FAPP, Ch. 2, p. 65.
- Graph Theory Activities from the Discrete Mathematics Project at [www.colorado.edu/education/DMP](http://www.colorado.edu/education/DMP)
- Class work, homework, quizzes, tests.

### Stage 3 – Learning Plan

**Learning Activities:**
- “Traversing Networks Investigation” activity from resource binder.
- “Muddy City Problem #2” activity from resource binder.
- “Temple of the Three Gods” activity from resource binder.
- “Multiples and Factors” activity from resource binder.
- practice exercises, pp. 259-269 from resource binder.
- practice exercises from FAPP, Ch. 2.
- NUMB3RS Activities found at [www.ti.com](http://www.ti.com)
Established Goals:
NJCCCS 4.2—All students will develop spatial sense and the ability to use geometric properties, relationships, and measurement to model, describe, and analyze phenomena.
NJCCCS 4.4—All students will be able to collect, organize, and display relevant data to answer questions that can be addressed with data; use appropriate statistical methods to develop and evaluate inferences and predictions that are based on data; and apply basic concepts of probability in order to make informed choices and reasonable decisions about information presented.
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Understandings:
Students will understand that...
- Discrete mathematics consists of tools and strategies for representing, organizing, and interpreting non-continuous data.
- Optimization is finding the best solution within given constraints.
- Grouping by attributes (classification) can be used to answer mathematical questions.
- Algorithms can effectively and efficiently be used to quantify and interpret discrete information.

Essential Questions:
- Is there a minimum number of colors we can use to color any map?
- How can graph coloring be used to solve real-life problems?

Students will know...
- the method for coloring maps so that no two sections bordering each other are the same color.
• the Four Color Theorem.
• the method for coloring vertex-edge graphs.
• relevant vocabulary: Four Color Theorem, vertex coloring, graph theory

_Students will be able to…_
• use vertex coloring techniques to solve conflicts, such as scheduling exams.
• color any map using the Four Color Theorem.
• color any diagram using the least number of colors.
STAGE 2 – Assessment Evidence

Performance Tasks:
- Create a map which can be colored using a given number of colors.
- Color the counties of New Jersey using the Four Color Theorem (resource binder).
- Color the states of the United States using the Four Color Theorem (resource binder).

Other Evidence:
- Applet exercises from FAPP, Ch. 3 found at www.whfreeman.com/fapp8e
- Writing projects from FAPP, Ch. 3, p. 101.
- Graph Theory Activities from the Discrete Mathematics Project at www.colorado.edu/education/DMP
  - Class work, homework, quizzes, tests.

Stage 3 – Learning Plan

Learning Activities:
- “Colorful Kites” activity from resource binder.
- “NJ and Southeast US” activity from resource binder.
- “Countries of Europe” from resource binder.
- “South America” activity from resource binder.
- “Coloring Graphs” activity from resource binder.
- Practice exercises, pp. B11-B14 from resource binder.
- Practice exercises from FAPP, Ch. 3.
- NUMB3RS Activities found at www.ti.com
DISCRETE MATHEMATICS
Unit 6—Scheduling
STAGE 1 – Desired Results

Established Goals:
NJCCCS 4.2—All students will develop spatial sense and the ability to use geometric properties, relationships, and measurement to model, describe, and analyze phenomena.
NJCCCS 4.4—All students will be able to collect, organize, and display relevant data to answer questions that can be addressed with data; use appropriate statistical methods to develop and evaluate inferences and predictions that are based on data; and apply basic concepts of probability in order to make informed choices and reasonable decisions about information presented.
NJCCCS 4.5—All students will use the mathematical processes of problem solving, communicating, reasoning, representing, and technology to gain deeper understanding of mathematical concepts, problem solving, and how to communicate mathematical ideas.
NJCCCS 3.2—All students will write in clear, concise, organized language that varies in content and form for different audiences and purposes.
NJCCCS 8.1—All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaboratively to create and communicate knowledge.

Understandings:
Students will understand that...
- Discrete mathematics consists of tools and strategies for representing, organizing, and interpreting non-continuous data.
- Optimization is finding the best solution within given constraints.
- Grouping by attributes (classification) can be used to answer mathematical questions.
- Algorithms can effectively and efficiently be used to quantify and interpret discrete information.

Essential Questions:
- Why is it necessary for governments and manufacturing firms to minimize their costs and operational problems?
- How can you determine the most efficient way to provide services on time?

Students will know...
- directed graphs can be used to solve efficiency problems.
• vertex-edge graphs can be used to solve conflicts.
• the Critical Path Algorithm.
• strategies for solving Sudoku puzzles.
• relevant vocabulary: directed graphs, conflicts, Critical Path Algorithm, Sudoku

Students will be able to…
• use a precedence relation to create a directed graph.
• schedule a job using the critical path algorithm and a specified number of processors.
• solve Sudoku puzzles of increasing difficulty.
STAGE 2 – Assessment Evidence

Performance Tasks:
• “Scheduling Classes” activity from resource binder (Discrete Mathematics Project)
• “Checker Tournament” activity from resource binder (Discrete Mathematics Project)
  • Write explicit instructions on how to make a peanut butter and jelly sandwich. The next day, teacher attempts to follow the directions and make a sandwich.

Other Evidence:
• Applet exercises from FAPP, Ch. 3 found at www.whfreeman.com/fapp8e
• Writing projects from FAPP, Ch. 3, p. 101.
• Graph Theory Activities from the Discrete Mathematics Project at www.colorado.edu/education/DMP
  • Class work, homework, quizzes, tests.

Stage 3 – Learning Plan

Learning Activities:
• “Chemicals” activity from resource binder.
• “High School Clubs” activity from resource binder.
• “Sounds Fishy to Me” activity from resource binder.
• “Coloring Graphs” activity from resource binder.
• Practice exercises, pp. 301-309 from resource binder.
• Practice exercises from FAPP, Ch. 3.
• NUMB3RS Activities found at www.ti.com
Established Goals:
NJCCCS 4.2—All students will develop spatial sense and the ability to use geometric properties, relationships, and measurement to model, describe, and analyze phenomena.
NJCCCS 4.3—All students will understand how patterns, relations, and functions are interrelated; be able to represent and analyze mathematical situations and structures using algebraic symbols; use mathematical models to understand quantitative relationships; and analyze change in various contexts.
NJCCCS 4.4—All students will be able to collect, organize, and display relevant data to answer questions that can be addressed with data; use appropriate statistical methods to develop and evaluate inferences and predictions that are based on data; and apply basic concepts of probability in order to make informed choices and reasonable decisions about information presented.
NJCCCS 4.5—All students will use the mathematical processes of problem solving, communication, connections, reasoning, representations and technology to gain deeper understanding of mathematical concepts, problem solving and how to communicate mathematical ideas.
NJCCCS 3.2—All students will write in clear, concise, organized language that varies in content and form for different audiences and purposes.
NJCCCS 8.1—All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaboratively to create and communicate knowledge.

Understandings:
Students will understand that...
- Discrete mathematics consists of tools and strategies for representing, organizing, and interpreting non-continuous data.
- Optimization is finding the best solution within given constraints.
- Grouping by attributes (classification) can be used to answer mathematical questions.
- Mathematical models can be used to describe and quantify physical relationships.

Essential Questions:
- Where is the Fibonacci sequence evident in nature?
- How have civilizations used the golden ratio and the golden rectangle?
**Students will know…**

- the recursive patterns embedded in the Fibonacci sequence.
- symmetry also includes the notions of balance, similarity and repetition, besides meaning mirror-image correspondence between parts of an object.
- the golden ratio or rectangle is a specific numerical proportion considered by the Greeks to be essential to beauty and symmetry.
- relevant vocabulary: symmetry, recursion, golden ratio, golden rectangle

**Students will be able to…**

- describe the recursive patterns embedded in the Fibonacci sequence.
- recognize and use the golden ratio and golden rectangle.
STAGE 2 – Assessment Evidence

Performance Tasks:
- “Breeding Rabbits” activity from resource binder (Discrete Mathematics Project)
- “The Golden Ratio” activity from resource binder (Discrete Mathematics Project)
- Find two pictures in a magazine (model and building). Prove that the proportions are the golden rectangle.

Other Evidence:
- Writing projects from FAPP, Ch. 19, p. 645.
- Graph Theory Activities from the Discrete Mathematics Project at www.colorado.edu/education/DMP
  - Class work, homework, quizzes, tests.

Stage 3 – Learning Plan

Learning Activities:
- View the video “Donald in Math Magicland”.
- Practice exercises, pp. 637-639 from resource binder.
- Practice exercises from FAPP, Ch. 19.
- NUMB3RS Activities found at www.ti.com
Established Goals:
NJCCCS 4.2—All students will develop spatial sense and the ability to use geometric properties, relationships, and measurement to model, describe, and analyze phenomena.
NJCCCS 4.3—All students will understand how patterns, relations, and functions are interrelated; be able to represent and analyze mathematical situations and structures using algebraic symbols; use mathematical models to understand quantitative relationships; and analyze change in various contexts.
NJCCCS 4.5—All students will use the mathematical processes of problem solving, communication, connections, reasoning, representations and technology to gain deeper understanding of mathematical concepts, problem solving and how to communicate mathematical ideas.
NJCCCS 3.2—All students will write in clear, concise, organized language that varies in content and form for different audiences and purposes.
NJCCCS 8.1—All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaboratively to create and communicate knowledge.

Understandings:
Students will understand that...
- Spatial sense and geometric relationships are a means to solve problems and make sense of a variety of phenomena.
- Shape and area can be conserved during mathematical transformations.
- Patterns and relationships can be represented graphically, numerically, symbolically, or verbally.
- Physical models can be used to clarify mathematical relationships.
- Tessellations can be created from a variety of different polygons.

Essential Questions:
- Where are transformations and tessellations evident in the real-world?
- What is the importance of M. C. Escher’s work?

Students will know...
- the methods for performing transformations on geometric figures (reflections, rotations, translations).
- tessellations are formed by using repeated shapes to cover a flat surface.
- not all regular polygons will tessellate.
Tangram is an ancient Chinese puzzle consisting of seven geometric pieces. Origami is the Japanese art of paper folding.

**Students will be able to…**

- recognize and describe transformations.
- use transformations in creating design patterns.
- create and identify regular tessellations (tilings)
- identify work by M.C. Escher
- create an Escher-like tessellation.
- solve tangram problems
- fold origami figures.
STAGE 2 – Assessment Evidence

Performance Tasks:
- Create an original Escher-like tessellation.
- Create a given picture using tangrams.
- Create an origami swan.

Other Evidence:
- Graph Theory Activities from the Discrete Mathematics Project at www.colorado.edu/education/DMP
- Class work, homework, quizzes, tests.

Stage 3 – Learning Plan

Learning Activities:
- “Tessellation Problems” activities from resource binder.
- “Alphabet Symmetry” activity from resource binder.
- “Flag Symmetry” from resource binder.
- “Geometric Shape Symmetry” from resource binder.
- “Tessellating Regular Polygons” from resource binder, using geometry templates.
- practice exercises from FAPP, Ch. 20.
- NUMB3RS Activities found at www.ti.com
Established Goals:
NJCCCS 4.2—All students will develop spatial sense and the ability to use geometric properties, relationships, and measurement to model, describe, and analyze phenomena.
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Understandings:
Students will understand that...

- Spatial sense and geometric relationships are a means to solve problems and make sense of a variety of phenomena.
- Patterns and relationships can be represented graphically, numerically, symbolically, or verbally.
- Physical models can be used to clarify mathematical relationships.
- Algorithms provide step-by-step directions for completing a task or solving a problem.

Essential Questions:
- What objects in nature exhibit a fractal-like structure?

Students will know...
- the iterative process of creating and recognizing fractal patterns.
- fractals form the basis for a method of image compression.
- fractals are built through the use of recursive stages.
- relevant vocabulary: fractal, iteration, Koch Snowflake, Sierpinski’s Triangle
Students will be able to…

• recognize and describe the iterative process of building fractals.
• create and describe the Koch Snowflake.
• create and describe Sierpinski’s Triangle.
• create and describe fractals based on trees.
STAGE 2 – Assessment Evidence

Performance Tasks:
• Create Sierpinski’s Triangle and complete “Math Questions on Sierpinski’s Triangle” (from resource binder).
• Create a personalized Spiralateral.

Other Evidence:
• Graph Theory Activities from the Discrete Mathematics Project at www.colorado.edu/education/DMP
  • Class work, homework, quizzes, tests.

Stage 3 – Learning Plan

Learning Activities:
• Resource Book (Section 5) activities from resource binder.
• “Generating Fractals” activities from resource binder.
• “Properties of Fractals” activities from resource binder.
• “Growing Tree” activity from resource binder.
• “Trees” activity from resource binder.
• Koch Snowflake” activity from resource binder.
• “Sierpinski’s Triangle and Variations” activity from resource binder.
• practice creating spirilaterals using dot paper.
• practice exercises from FAPP, Ch. 20, pp. 644-645.
• NUMB3RS Activities found at www.ti.com
Established Goals:
NJCCCS 4.1—All students will understand the meaning of numbers, how they may be represented and the relationships among them. They will perform computations and acquire knowledge of the physical world from the point of view of quantitative relationships.
NJCCCS 4.5—All students will use the mathematical processes of problem solving, communication, connections, reasoning, representations and technology to gain deeper understanding of mathematical concepts, problem solving and how to communicate mathematical ideas.
NJCCCS 3.2—All students will write in clear, concise, organized language that varies in content and form for different audiences and purposes.
NJCCCS 8.1—All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaboratively to create and communicate knowledge.
NJCCCS 9.2—All students will develop skills and strategies that promote personal and financial responsibility related to financial planning, savings, investment, and charitable giving in the global economy.
NJCCCS 9.3—All students will apply knowledge about and engage in the process of career awareness, exploration and preparation in order to navigate the globally competitive work environment of the information age.

Understandings:
Students will understand that...
- Each job, career and profession has a set of preparation requirements, career exploration experiences and different opportunities for personal and professional growth and satisfaction.
  - In the 21st century, people will most likely have multiple careers and jobs.
  - Personal actions today and tomorrow may have an effect on future employment.
  - Choices we make as individuals affect self, family, community and the world.
  - Financial choices have costs, benefits and consequences.

Essential Questions:
- How do I decide what I want to be and how do I prepare for my career?
- Why do I need a career plan?
- When should I start planning financially, and how is it best to do so?
Students will know…
- Different methods for computing interest
- Different models for saving money.
- Different models for borrowing money.
- The importance of budgeting in financial planning.
- Relevant vocabulary: savings, credit, interest

Students will be able to…
- Compute simple and compound interest.
- Describe different methods of saving money.
- Describe different methods of borrowing money.
- Create a budget for living independently.
STAGE 2 – Assessment Evidence

Performance Tasks:
- “Moving Out”—students research careers and develop a plan to be able to live independently (from www.salary.com)

Other Evidence:
- Applet exercises from FAPP, Ch. 21 & 22 found at www.whfreeman.com/fapp8e
- Writing projects from FAPP, Ch. 21 & 22, pp. 707, 729.
- Class work, homework, quizzes, tests.

Stage 3 – Learning Plan

Learning Activities:
- practice activities pp. 699-728 from resource binder.
- practice exercises from FAPP, Ch. 21 & 22, pp. 700-706, 725-728.
- NUMB3RS Activities found at www.ti.com
Established Goals:
NJCCCS 4.3—All students will understand how patterns, relations, and functions are interrelated; be able to represent and analyze mathematical situations and structures using algebraic symbols; use mathematical models to understand quantitative relationships; and analyze change in various contexts.
NJCCCS 4.4—All students will be able to collect, organize, and display relevant data to answer questions that can be addressed with data; use appropriate statistical methods to develop and evaluate inferences and predictions that are based on data; and apply basic concepts of probability in order to make informed choices and reasonable decisions about information presented.
NJCCCS 4.5—All students will use the mathematical processes of problem solving, communication, connections, reasoning, representations and technology to gain deeper understanding of mathematical concepts, problem solving and how to communicate mathematical ideas.
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Understandings:

Students will understand that...

• Discrete mathematics consists of tools and strategies for representing, organizing, and interpreting non-continuous data.
• Grouping by attributes (classification) can be used to answer mathematical questions.
• Patterns and relationships can be represented graphically, numerically, symbolically, or verbally.
• Mathematical models can be used to describe and quantify physical relationships.
• The message conveyed by the data depends on how the data is collected, represented, and summarized.

Essential Questions:

• Why are codes important in society?
<table>
<thead>
<tr>
<th><strong>Students will know</strong>…</th>
</tr>
</thead>
<tbody>
<tr>
<td>• a variety of ways to encrypt information (alpha-numeric codes, bar codes, ISBN numbers, vin numbers).</td>
</tr>
<tr>
<td>• strategies for decoding information.</td>
</tr>
<tr>
<td>• relevant vocabulary: secret codes, bar codes, ZIP codes, ISBN numbers, vin numbers, glyph, cryptoquote, pictogram</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Students will be able to</strong>…</th>
</tr>
</thead>
<tbody>
<tr>
<td>• decode encrypted information in a variety of different coding situations.</td>
</tr>
<tr>
<td>• create a coded message.</td>
</tr>
<tr>
<td>• create a glyph.</td>
</tr>
</tbody>
</table>
STAGE 2 – Assessment Evidence

Performance Tasks:
  • Using different ciphers, write a note to your army and decode before getting captured.
  • Create a personalized glyph. Have another student try to decode the information contained in the glyph.
  • Media Scavenger Hunt—decode a message and use the internet to find the answer.

Other Evidence:
  • Writing projects from FAPP, Ch. 16 & 17, pp. 530, 567.
  • Graph Theory Activities from the Discrete Mathematics Project at www.colorado.edu/education/DMP
  • Class work, homework, quizzes, tests.

Stage 3 – Learning Plan

Learning Activities:
  • “Codes” activity from resource binder.
  • “Pictogram” activities from resource binder.
  • practice exercises, pp. 526-530 from resource binder.
  • practice exercises from FAPP, Ch. 16 & 17.
  • NUMB3RS Activities found at www.ti.com
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Understandings:
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- Discrete mathematics consists of tools and strategies for representing, organizing, and interpreting non-continuous data.
- Grouping by attributes (classification) can be used to answer mathematical questions.
- Patterns and relationships can be represented graphically, numerically, symbolically, or verbally.
- Mathematical models can be used to describe and quantify physical relationships.
- Algorithms and visual tools can be used to solve problems and answer real-world questions, such as game strategies.
- Optimization is finding the best solution within given constraints.

Essential Questions:
• Why is game theory helpful?
• How can game theory be applied in practical situations in other disciplines?
Students will know…
• game theory uses mathematical tools to study situations involving both conflict and cooperation.
  • strategies chosen by players have outcomes which are the consequences of choices made during the playing of a game.
  • a gain for one player means a loss for another.
• relevant vocabulary: conflict, strategy, outcome, game theory

Students will be able to…
• devise strategies for winning two-person games.
  • devise strategies for winning simple games and express those strategies as sets of directions.
STAGE 2 – Assessment Evidence

Performance Tasks:
• Create an original game, including rules for playing and a game board or game pieces.

Other Evidence:
• Writing projects from FAPP, Ch. 15, pp. 505.
• Graph Theory Activities from the Discrete Mathematics Project at www.colorado.edu/education/DMP
  • Class work, homework, quizzes, tests.

Stage 3 – Learning Plan

Learning Activities:
• “Mathematical Games” activities from resource binder.
• “Star Slide Seven” game from resource binder.
• “Land on 0” game from resource binder.
• “Matchings and Games” activities from resource binder.
• “Some Hex Grids” game from resource binder.
• “Make 21” game from resource binder.
• “Games and Strategies” activities from resource binder.
• “Las Vegas Casino” game from resource binder.
• “Resource Book (Section 12)” activities from resource binder.
• “Resource Book (Section 13)” activities from resource binder.
• practice exercises, pp. 500-504 from resource binder.
• practice exercises from FAPP, Ch. 15.
• NUMB3RS Activities found at www.ti.com
# DISCRETE MATHEMATICS

## PACING GUIDE

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